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Basic Circuit Elements and Waveforms

1.1 INTRODUCTION

The phenomenon of transferring charge from one point in a circuit to another is termed as "electric current". An electric current may be defined as the rate of net motion of electric charge q across a cross-sectional boundary. Random motion of electrons in a metal does not constitute a current unless there is a net transfer of charge. Since the electron has a charge of 1.6021×10^{-19} coulomb, it follows that a current of 1 ampere corresponds to the motion of $1/(1.6021 \times 10^{-19}) = 6.24 \times 10^{18}$ electrons per second in any cross-section of a path.

***André-Marie Ampère** (1775–1836), a French mathematician and physicist, was born in Lyon, France. At that time the best mathematical works were in Latin and Ampere was keenly interested in mathematics, so he at the age of 12 mastered in Latin in a few weeks. He was a brilliant scientist and a prolific writer. He formulated the laws of electromagnetics. He invented the electromagnet and ammeter. The unit of electric current, was named in his honour, the ampere.*

In terms of the atomic theory concept, an electric current in an element is the time rate of flow of free electrons in the element. The materials may be classified as (*i*) conductors, where availability of free electrons is very large, as in the case of metals; (*ii*) insulators, where the availability of free electrons is rare, as in the case of glass, mica, plastics, etc. Other materials, such as germanium and silicon, called semiconductors, play a significant role in electronics. Thermally generated electrons are available as free electrons at room temperature, and act as conductors, but at 0 Kelvin they act as insulator. Therefore, conductivity is the ability or easiness of the path or element to transfer electrons. The resistivity of the path is the resistance offerred to the passage of electrons, *i.e.*, resistance (resistivity) is the inverse of conductance (conductivity).

In circuit analysis, we are concerned with the four basic manifestations of electricity, namely, electric charge $q(t)$, magnetic flux $\phi(t)$, electric potential $v(t)$ and electric current $i(t)$. We assume that the reader is familiar with these concepts. There are four fundamental equations of circuit analysis.

The current[1] through a circuit element is the time derivative of the electric charge $q(t)$ *i.e.*,

$$i(t) = \frac{d}{dt} q(t) \tag{1.1}$$

The unit of charge is coulomb. The unit of current $i(t)$ is coulomb per second, which is termed ampere (abbreviated A) in honour of the French physicist Andrè-Marie Ampère (1775–1836).

The potential difference between the terminals of a circuit element in a magnetic field is equal to the time derivative of the flux $\phi(t)$, *i.e.,*

$$v(t) = \frac{d}{dt} \phi(t) \tag{1.2}$$

The unit of potential is webers per second, which is called volts in honour of the Italian physicist Alessandro Volta (1745–1827).

Alessandro Giuseppe Antonio Anastasio Volta *(1745–1827), an Italian physicist, was born in a noble family in Como, Italy. Volta was engaged himself in performing electrical experiments at the age of 18. His invention of the electric cell (battery) in 1796 revolutionised the use of electricity. Volta received many honours during his lifetime. The unit of voltage or potential difference, was named in his honour, the volt.*

Voltage is the energy required to move 1 coulomb of charge through an element, *i.e.*,

$$v(t) = \frac{dw}{dq} \tag{1.3}$$

The instantaneous power $p(t)$ delivered to a circuit element is the product of the instantaneous value of voltage $v(t)$ and current $i(t)$ of the element.

$$p(t) = v(t)\, i(t) \tag{1.4}$$

The unit of power is watt in honour of the British inventor James Watt (1738–1819).

The energy delivered to a circuit element over the time interval (t_0, t) is given by

$$E(t_0, t) = \int_{t_0}^{t} p(x)\, dx = \int_{t_0}^{t} v(x)\, i(x)\, dx \tag{1.5}$$

The unit of energy is watt-second, which is called joule in honour of the British scientist J.P. Joule (1818–1889).

Equation (1.1) through (1.5) hold for all circuit elements, irrespective of their nature. However, the relation between the voltage and current of an element depends entirely on the physical nature of the circuit element. The circuit element may be linear, non-linear and time-varying or time-invariant.

[1]From French word *intensité*, current symbol is taken as i.

1.2 CIRCUIT COMPONENTS

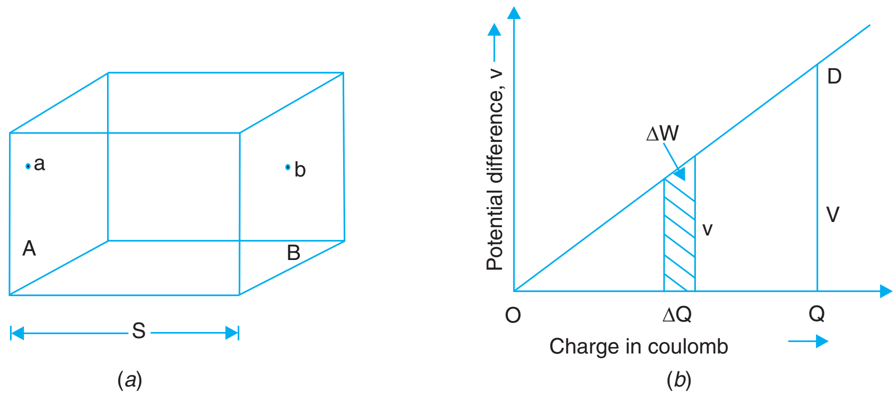

Fig. 1.1. (*a*) Energy storage on a capacitor (*b*) Relation between energy and stored charge

1.2.1 Capacitance

Consider two identical plates, both without a net charge, separated by a distance '*s*' as in Fig. 1.1 (*a*). If a small positive charge, ΔQ is removed from '*a*' of plate *A* and taken to '*b*' of plate *B,* plate *A* has a negative charge and plate *B* has a positive charge. Work is done in moving ΔQ from *A* to *B* because there is a force on ΔQ trying to return ΔQ to *A*. Accordingly, plate *B* now has a positive potential with respect to plate *A*. When a second small positive charge of same magnitude, ΔQ, is taken from *A* to *B*, the force is larger and the work done greater. The process is linear if ΔQ is sufficiently small. Hence, we can say that the charge *Q* is proportional to the potential difference *V*, *i.e.*,

$$Q \propto V$$

or $$Q = CV \text{ coulombs}$$

or $$C = Q/V$$

The unit of capacitance is farad in honour of Michael Faraday. It is defined as the ratio, coulomb per volt. The physical device that permits storage of charge is a capacitor and its ability to store charge is its capacitance. The reciprocal of capacitance is defined as elastance.

The energy expended at any point, in the process of transferring charge from one plate to the other, is shown in the shaded region of Fig. 1.1 (*b*). Therefore, the total energy required to transfer *Q* coulombs of charge, resulting in a final potential of *V* volts between the plates, is the area of the triangle *OQD*, *i.e.*,

$$W = \frac{1}{2} VQ = \frac{1}{2} V(CV) = \frac{1}{2} CV^2 \text{ joules} \quad (1.6)$$

The energy is contained in the electric field and in the lines of force.

When two parallel plates of cross-sectional area A_1 with a spacing between them as s_1, and a battery of V_1 volt is connected to the capacitor plates as in Fig. 1.2 (*a*), then the battery draws *Q* coulombs of electrons from the left plate and adds *Q* coulombs of electrons to the right plate. Hence, the left plate has + Q and the right plate – Q charge. The charge density on the plate is σ. If the cross-sectional area alone is increased from A_1 to A_2 as in Fig. 1.2 (*b*), then, as the charge density remains unchanged, the total charge on A_2 must have increased. That is, C, the capacitance, is proportional to the cross-sectional area A. Now, if the spacing has changed from s_1 to s_2, where $s_2 > s_1$, without altering other quantities, *i.e.*, A_1 and V_1, as in Fig. 1.2 (*c*), the charges + Q and – Q on the plates cause an attraction between them. If the plates are moved apart to a new separation s_2, more work has to be done to move these charges apart, if the charges on the plates are to be maintained at + Q and – Q, that is, the voltage must be increased. This increase in voltage (or work) is proportional to the increase in spacing s.

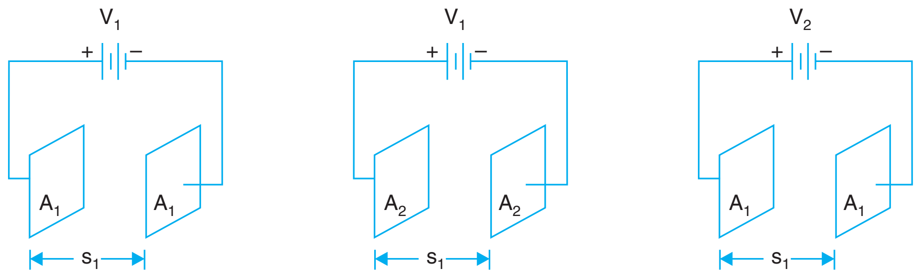

Fig. 1.2. Parallel plate capacitors showing different plate spacing and plate area

So, $C \propto A$ and $C \propto 1/s$

Hence $C \propto A/s$

or $C = \varepsilon_0 A/s = 8.854\, A/s$ pF

where, A is in square metres and s in metres. The above discussion is for vacuum having dielectric constant ε_0. Let us define ε, the dielectric constant

$$\varepsilon = C_2/C_1$$

where, C_1 and C_2 are the capacitance values in vacuum and in the new dielectric material respectively. Then, the capacitance of parallel plate capacitor in a dielectric medium of dielectric constant ε, in SI units, can be written as

$$C = \varepsilon\varepsilon_0\, A/s \text{ farad} = 8.854\, \varepsilon\, A/s \text{ pF} \tag{1.7}$$

as $\varepsilon_0 = 8.854 \times 10^{-12}$

These equations are valid for parallel plate capacitors. For other geometric shapes such as two spheres, two conductors, or one conductor and the ground, the formula would be different. In each case C will be a function of the geometry of the conductors and ε.

Various materials such as mica, paper and oil are some of the dielectrics used. Typical values of dielectric constants are listed in Table 1.1. It may be observed that in actual capacitors, electric field lines are not parallel and fringing occurs at the edges. With fringing, we have additional electric field lines. These cause the actual capacitance value to be somewhat larger than the value obtained from Eqn. (1.7). As long as A and s are large, the fringing effect can be neglected.

Table 1.1 Typical Values of Dielectric Constants

Material	*Dielectric constant*
Vacuum	1
Dry air	1.0006
Glass	5 to 10
Mica	5 to 7
Oil	3
Paper, paraffin	2.5
Porcelain	6
Rubber	3
Teflon	2
Titanium compound	500 to 5000
Water	80

Sheets of metal foil separated by strips of mica can withstand high voltages and have excellent characteristics at radio frequencies.

Ceramic capacitors are manufactured by depositing directly on each side of a ceramic dielectric, the silver coatings that serve as capacitor plates. These capacitors have a very high capacitance per unit volume.

Paper capacitors are manufactured by winding long narrow sheets of alternate layers of aluminium foil and wax-impregnated paper into compact rolls. Large paper capacitors are often enclosed in a can filled with special oils. These oils improve the breakdown characteristics of the dielectric.

Electrolytic capacitors can only be used in a circuit that maintains the polarity of the voltage in one direction. If the voltage is reversed, the capacitor acts as a short-circuit. An aluminium electrode serves as the positive plate and an alkaline electrolyte serves as the negative plate. A very thin aluminium oxide film forms on the surface of the positive plate to serve as the dielectric. These capacitors have high values of capacitance, ranging from 1 to 2 μF to the order of several thousand microfarads.

Capacitive Circuit

The potential difference v between the terminals of a capacitor is proportional to the charge q on it. We know

$$v \propto q$$

or

$$v = q/C \tag{1.8}$$

where, C is the constant of proportionality and is called the capacitance.

Now,

$$i = \frac{dq}{dt} = C\left(\frac{dv}{dt}\right) \tag{1.9}$$

$$\int dv = \frac{1}{C}\int i\,dt$$